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Variance Estimation for Measures of Change

S.A.M.P.L.E. CONFERENCE

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Evolution of Measures of Poverty and Income Inequality Indicators in the European Union

Advanced Methodology for European Laeken Indicators

- Project is funded by the European Commission within the seventh Framework Programme
- Social Sciences and Humanities
  Area 6.2 – Developing better indicators for policy
- DG RTD in cooperation with DG ESTAT
- Project officer: Dr. Ian Perry
- EC contribution 1.089 M€

Co-ordinator: Ralf Münnich (muennich@uni-trier.de)
Homepage: http://ameli.surveystatistics.net

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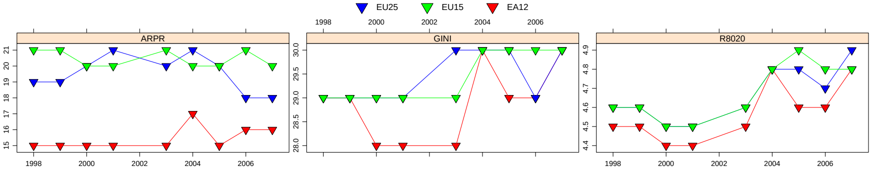
Aim of EU-SILC

To monitor the process towards agreed policy goals we are interested in the evolution of social indicators.



Source: Eurostat, 2009, http://epp.eurostat.ec.europa.eu/portal/page/portal/statistics/search_database

- Reading naively point estimator tables may lead to over-interpret the data.
- Was the change (in time) of an indicator value significant or not?
- How to *measure* significant changes of ARPR, GINI, and QSR?

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Two Problems arise:

1. The statistics in question (the Laeken indicators) are highly non-linear.
   - Basic variance estimation formulas cannot be applied directly.
2. The Surveys used to estimate the indicator values (EU-SILC) are often time dependent.
   - The correlation through time between indicators has to be taken into account.

Dell and d'Haultfoeuille (2007)

Goga, Deville, and Ruiz-Gazen (2009, Biometrika)

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Variance estimation for non-linear statistics

- Resampling methods
  Kovačević and Yung (1997)
  - Balanced repeated replication
  - Jackknife
  - Bootstrap
- Linearization methods
  - Taylor's method
  - Woodruff linearization
    Woodruff (1971) or Andersson and Nordberg (1994)
  - Estimating equations
    Kovačević and Binder (1997)
  - Influence functions
    Deville (1999)

Application to poverty and inequality indicators

Using the linearized values for the statistics ARPR, GINI, and QSR to approximated there variance:

$$\mathsf{V}(\widehat{\mathcal{I}}) \approx \mathsf{V}\Big(\sum_i \frac{1}{\pi_i} \cdot u_i\Big)$$

If the weights used in estimating $\mathcal{I}$ are obtained by a calibration of design weights, u_i are the residuals of the regression of the linearized values on the auxiliary variables used in the calibration, (cf. Deville, 1999).

Indicator $\mathcal{I}$	Source
ARPR:	Deville (1999), Osier (2009)
GINI:	Kovačević and Binder (1997)
QSR:	Hulliger and Münnich (2007)

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Ratio in time

$$\widehat{V}(\widehat{\mathcal{R}}_{\mathcal{I}}) = \widehat{V}(\widehat{\mathcal{I}}_{t_1}/\widehat{\mathcal{I}}_{t_0})$$
$$= \frac{1}{\widehat{\mathcal{I}}_{t_0}^2} \cdot \left(\widehat{\mathcal{R}}_{\mathcal{I}}^2 \cdot \widehat{V}(\widehat{\mathcal{I}}_{t_0}) + \widehat{V}(\widehat{\mathcal{I}}_{t_1}) - 2 \cdot \widehat{\mathcal{R}}_{\mathcal{I}} \cdot \widehat{Cov}(\widehat{\mathcal{I}}_{t_0}, \widehat{\mathcal{I}}_{t_1})\right)$$

Covariance estimation for non-linear statistics

$$\widehat{Cov}(\widehat{\mathcal{I}}_{t_0}, \widehat{\mathcal{I}}_{t_1}) = \widehat{Cov}\Big(\sum_{i \in S_{t_0}} \frac{u_i}{\pi_i}, \sum_{j \in S_{t_1}} \frac{u_j}{\pi_j}\Big)$$
$$= \sum_{i \in S_{t_0}} \sum_{j \in S_{t_1}} \left(1 - \frac{\pi_i \cdot \pi_j}{\pi_{ij}^*}\right) \cdot \frac{u_i}{\pi_i} \cdot \frac{u_j}{\pi_j}$$

$\pi_i = P(i \in S_{t.})$; $\quad \pi_{ij}^* = P(i \in S_{t_0}, j \in S_{t_1})$

Rotational samples in EU-SILC

Survey Year					
Y – 1	S_4^{Y-4}	S_3^{Y-3}	S_2^{Y-2}	S_1^{Y-1}	
Y – 0		S_4^{Y-3}	S_3^{Y-2}	S_2^{Y-1}	S_1^{Y-0}
	Y – 4	Y – 3	Y – 2	Y – 1	Y – 0

Selection Year

Actual Sampling Plan The population is partitioned into a rotational panel with 4 rotation groups (quarters).

- S_{CS}^{Y-1} A stratified sample drawn independently from groups U^{Y-4}, U^{Y-3}, U^{Y-2}, and U^{Y-1}.
- S_{CS}^{Y-0} A stratified sample drawn from U^{Y-4} plus the units in S_{CS}^{Y-1} without the units in S_4^{Y-4}, (assumes a static population).

Households as PSUs

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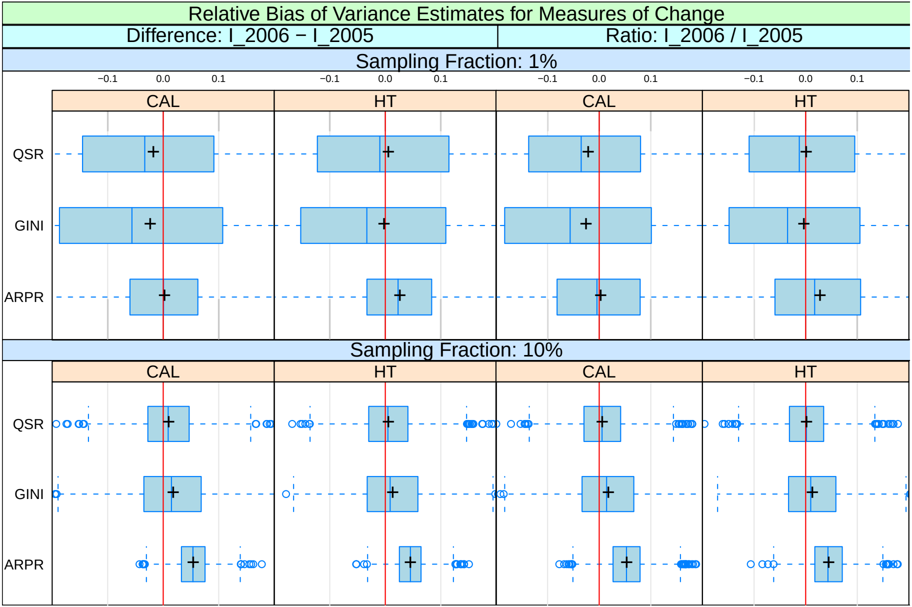

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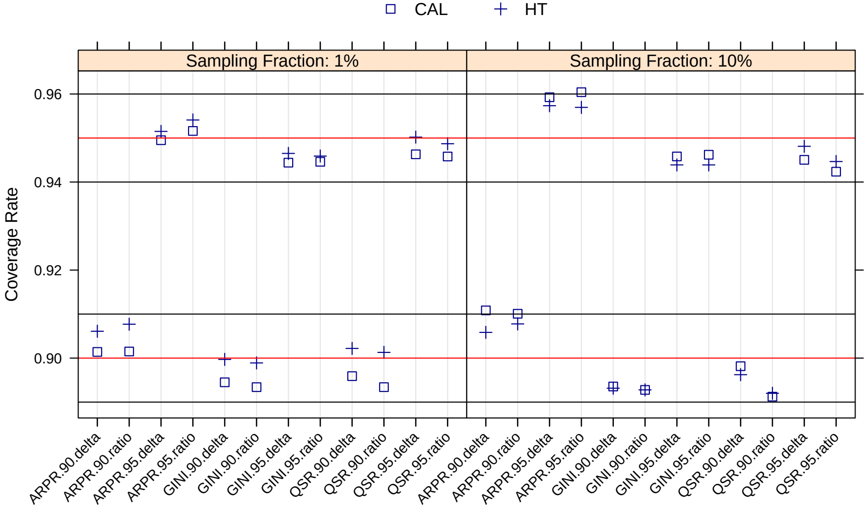
Coverage Rates for the Different Sampling Fractions



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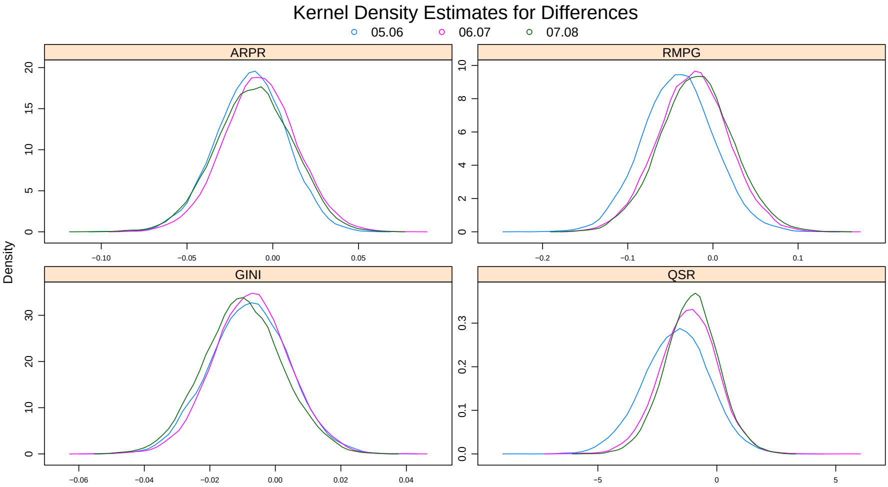

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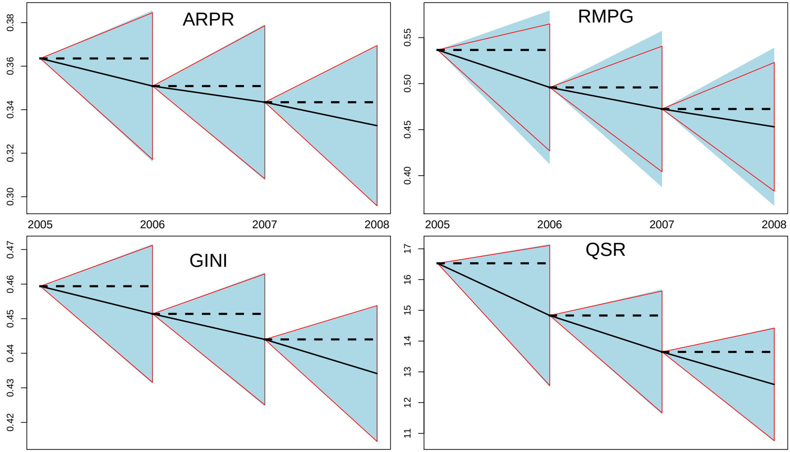
Test of Significant Change: $H_0 : \Delta = \Delta_0 = 0$



Hulliger (2005), Displays of indicators and of their accuracy, Conference on Visualising and Presenting Indicator Systems

Summary and outlook

- ARPR is less sensitive towards skewed distributions but more tends to be biased (density estimtaion)
- GINI and QSR are relatively non-robust against very skewed distributions
- Next steps
  - Non-linear calibration (on GINI or quantiles)
  - Estimation of the covariance between estimated totals in more complex dependent sampling surveys (Berger, 2004)
  - Introduction of robust methods for GINI and QSR